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THE ROYAL SOCIETY

Magnetic reconnection and some solar applications

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Magnetic reconnection is a fundamental plasma-physics process that is of great importance for the Sun, the Earth's magnetosphere and all astrophysical objects which have magnetic fields. It is a process central to the generation of magnetic fields in stars and also plays a major role in the heating of solar and stellar coronae. The development of both large (e.g. flares) and small (e.g. coronal bright points) dynamic phenomena observed on the Sun depends on reconnection and it is likely that reconnection may also be important for the acceleration of solar and stellar winds.

It is 50 years since the first seeds of ideas for magnetic reconnection were sown and over 40 years since the classic Sweet–Parker mechanism was suggested. Since then the majority of the research has focused on reconnection in two dimensions. However, in the last few years attentions have turned to understanding the intricacies of reconnection in three dimensions. In this paper, the classical aspects of two-dimensional reconnection are reviewed, together with various mechanisms for reconnection in three dimensions, in particular, spine, fan and quasi-separatrix layer reconnection. The paper is then rounded off with examples of some solar phenomena where reconnection is believed to be present. In particular, the heating of some observed small-scale events in the corona is investigated and the question of quiet coronal heating due to nanoflares and microflares is addressed.

Keywords: Sun; magnetic fields; magnetic reconnection; corona; coronal heating

1. Introduction

The magnetic field of the Sun was first discovered by Hale (1908), who used the relatively new astronomical technique of spectroscopy. As his narrow slit moved across the Sun's surface the lines in his spectrograph began to broaden unmistakably and split as the slit crossed a sunspot. Then in the 1940s, Giovanelli (1946) suggested, in a *Nature* article, that magnetic nulls were the origin of solar flares. This paper pointed theorists along roads that have led to the discovery of magnetic reconnection, one of the most important fundamental plasma physics processes. In the last 50 years, not only has the concept of magnetic reconnection been developed, but considerable progress has been made in understanding the crucial role it plays, not only on the Sun, but also in the Earth's magnetosphere and more recently in many astrophysical phenomena.

The importance of magnetic reconnection in so many astrophysical processes comes about because it is the mechanism by which magnetic fields may globally change their

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topology and release stored energy. It can convert magnetic energy to both thermal and kinetic energy, as well as accelerate particles. It is also thought to create large electric currents, shock waves and filamentation.

Progress in understanding the importance of magnetic fields and, in particular, reconnection has been made both observationally and theoretically. For example, rocket flights through the Earth's magnetosphere have provided direct *in situ* measurements of plasma properties. Also remote sensing missions to view the Sun and other astrophysical objects have given us a global perspective of the different behaviour of magnetic phenomena. In particular, since the late 1960s and early 1970s, space observations of the corona in X-rays have dramatically revolutionized our ideas about the Sun's atmosphere. On the theoretical side, the last 50 years have seen advances in our understanding made by both analytical and computational approaches. Most notably the first tentative, but recognizable, ideas on magnetic reconnection were presented by Dungey (1953). Then Sweet (1958) and Parker (1957) produced order-of-magnitude estimates for reconnection in a diffusion layer. Later, Petschek (1964) took the next big step and suggested a mechanism for fast reconnection. Biskamp opened up the computational work with his controversial paper in 1986. Since then much work has been carried out on reconnection both in two and three dimensions.

In the following section (§ 2) I briefly describe the essentials of two-dimensional reconnection including both of the classical two-dimensional mechanisms, namely, Sweet–Parker reconnection and Petschek reconnection. Over the last 10 years, however, the focus has shifted towards the analysis of three-dimensional reconnection. In § 3, several possible mechanisms for reconnection in three dimensions are outlined. Then, in § 4, applications of reconnection mechanisms with a solar bias are considered. These include an explanation for the creation of small-scale events in the solar corona, known as coronal bright points. Also the question of whether small-scale events dominate the heating of the quiet corona is considered. Finally, in § 5, this review article is concluded.

2. Reconnection in two dimensions

(*a*) *Induction equation*

It is well known that reconnection can occur in two dimensions. Back in the 1950s, Dungey (1953) suggested that a potential X-type null, if perturbed, would collapse and induce current at the null provided that the field lines were free to move. This in itself, is of course, not reconnection since for reconnection to occur there must be diffusion of the field and not simply advection. For a quick reminder of advection and diffusion let us consider the *induction equation* which describes the evolution of a magnetic field, $\boldsymbol{B}$, affected by a velocity field, $\boldsymbol{v}$:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}, \tag{2.1}$$

where the diffusion coefficient $\eta = 1/(\mu_0 \sigma)$, μ_0 is the magnetic permeability and σ is the electrical conductivity. This equation comes from eliminating the electric field, $\boldsymbol{E}$, and electric current, $\boldsymbol{j}$, from the following three equations: Faraday's law ($\partial \boldsymbol{B}/\partial t = -\nabla \times \boldsymbol{E}$); Ampère's law ($\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j}$) and Ohm's law of the form

$$\boldsymbol{j} = \sigma(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}). \tag{2.2}$$

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Table 1. *Typical values for length-scales, velocity scales and magnetic Reynolds numbers for various astrophysical objects*

(Values of R_{m} are approximations.)

astrophysical object	L (m)	v (m s^{-1})	η (m^2 s^{-1})	R_{m}
solar corona	10^8	10^3	1	10^{11}
sunspot	10^7	10^3	8×10^2	1.3×10^7
Earth's magnetosphere[a]	10^7	10^5	10	10^{11}
accretion discs in CVs	10^8	10^3	1	10^{11}
accretion discs in AGN	10^8	10^3	3×10^{-2}	3×10^{13}
jets in CVs	10^7	10^5	10^4	10^8

[a]Kivelson & Russell (1995).

In most situations, Ohm's law is simplified with the left-hand side set to zero, since in the majority of the Universe $\boldsymbol{j}$ is negligible. However, it is possible that in localized regions, where the gradients of the magnetic field $\boldsymbol{B}$ are large, $\boldsymbol{j}$ may become significant. For such situations the above form for Ohm's law will normally suffice unless a more generalized Ohm's law is considered, including, for example, the Hall term.

The induction equation (2.1) has two terms on the right-hand side. The first term is known as the *advection term*. It represents the carrying of the field with the plasma. From Alfvén's theorem, it can be shown that if this term is dominant then the magnetic field is effectively frozen-into the plasma. That is to say, plasma elements can only move along the field and not across it. On the other hand, the second term on the right-hand side of (2.1), known as the *diffusion term*, describes the slippage of the magnetic field through the plasma. Only when this term becomes large can reconnection occur. In general, this term is negligible, since in most of the Universe $\boldsymbol{j}$ is tiny. However, where the gradients in the magnetic field become large enough, we find large electric currents, short length-scales and the diffusion term may dominate. Hence, large electric currents are essential for reconnection.

A useful parameter, known as the magnetic Reynolds number, R_{m}, represents the magnetic field's ability to advect or diffuse. It equals the ratio of the advection and diffusion terms of (2.1):

$$R_{\mathrm{m}} = \frac{|\nabla \times (\boldsymbol{v} \times \boldsymbol{B})|}{|\eta \nabla^2 \boldsymbol{B}|} = \frac{vL}{\eta},$$

where v and L are typical velocity and length-scales. Typical values for v, L, η and R_{m} for various different astrophysical objects are given in table 1. In general, the magnetic Reynolds number is very large and so, in most of the Universe, the magnetic field simply advects with the plasma. It is only in localized regions, for instance, near nulls in two dimensions, that steep gradients in the magnetic field induce currents and create localized diffusion regions.

In figure 1, a simple sketch demonstrates the essential elements of what can happen to magnetic field lines that pass through a diffusion region. The two field lines labelled AB (dashed) and CD (dotted) lie near an X-type null in topologically distinct regions. They are carried in towards the X-point where they lie along the field lines, called *separatrices*, that divide the topologically distinct regions. In the immediate vicinity

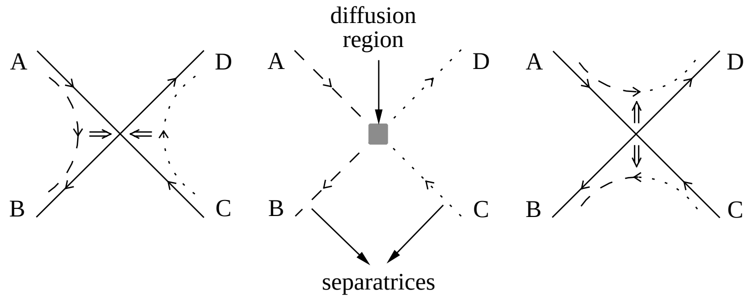


Figure 1. Sketch of two-dimensional reconnection at an X-point of field lines AB (dashed) and CD (dotted) to form lines AD and CB.

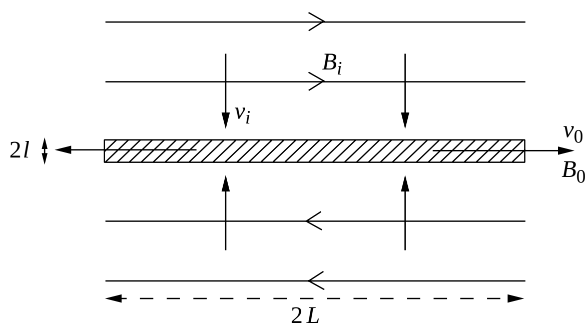


Figure 2. Sketch of a Sweet–Parker diffusion region and surrounding field.

of the null itself strong currents may be generated allowing diffusion to occur. In this region, called a diffusion region, reconnection can take place and new field line connections may be created. The new field lines, one from A to D and the other from C to B, are made up of halves of the old field lines and are then advected away from the diffusion region.

(*b*) *Sweet–Parker reconnection*

The first analytical investigation of magnetic reconnection in two dimensions was carried out by Sweet (1958) and Parker (1957). Their now famous classical reconnection mechanism, called Sweet–Parker reconnection, gave order-of-magnitude estimates for the behaviour of the plasma and field near a diffusion region. They considered a situation where the reconnection, assumed to be steady and incompressible, occurs in a long narrow diffusion layer (or current sheet), with length $2L$ and width $2l$. The diffusion layer divides two parallel, but oppositely directed, magnetic fields B_{i} (figure 2) which are carried towards the diffusion layer at an inflow rate of v_{i}. Since the reconnection is steady, v_{i} equals the rate of diffusion η/l. Also, from mass continuity, the amount of plasma entering the sheet must equal the amount leaving. Hence, since the system is incompressible, $v_{\mathrm{i}}L = v_{\mathrm{o}}l$, where v_{o} is the outflow speed of the plasma. Furthermore, from the equation of motion, it can easily be shown that this outflow speed must equal the inflow Alfvén speed, $v_{\mathrm{o}} = v_{\mathrm{Ai}} = B_{\mathrm{i}}/\sqrt{\mu\rho}$.

In two dimensions, when reconnection is steady, its rate can be measured by the rate of inflow of the magnetic field, v_{i}. However, a more useful dimensionless rate is

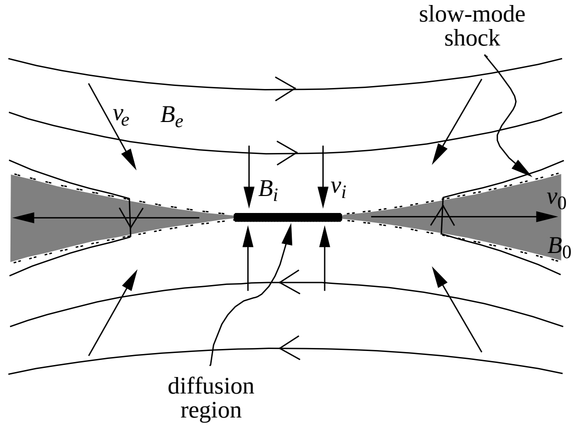


Figure 3. Sketch of Petschek's reconnection regime.

given by the Alfvén Mach number, $M_{\mathrm{Ai}} = v_{\mathrm{i}}/v_{\mathrm{Ai}}$. From the above relations for the Sweet–Parker current sheet, the rate of reconnection is found to be

$$M_{\mathrm{Ai}} = 1/\sqrt{R_{\mathrm{mi}}},$$

which is of the order of 10^{-6} if R_{mi} is about 10^{12}. This rate of reconnection is too slow to explain solar flares, if classical values for the magnetic Reynolds number are used, and therefore Sweet–Parker reconnection is known as slow reconnection.

(*c*) *Petschek reconnection*

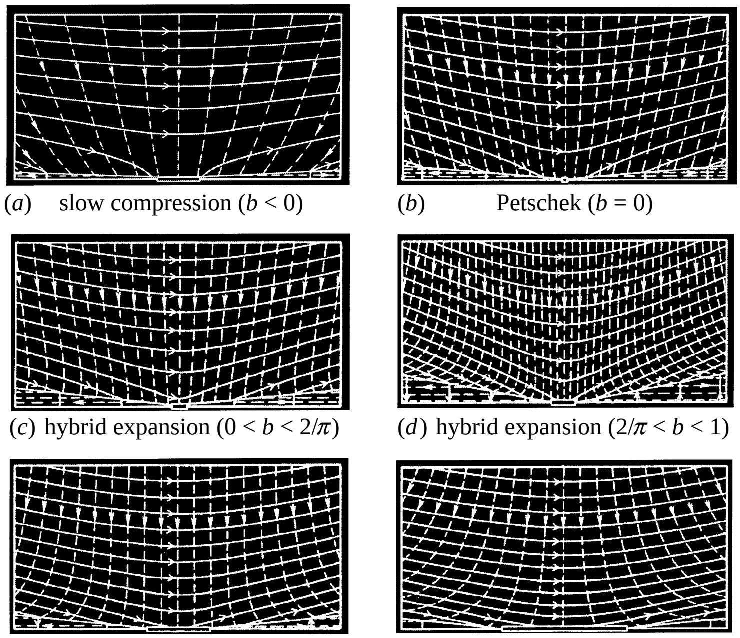
Six years later, Petschek (1964) presented a new reconnection mechanism. He realized that, if the inflow into the diffusion region is fast enough, then the diffusion region would act like an obstacle in the flow and create shocks. Of course, for steady reconnection the inflow speed is limited by the rate of diffusion and so Petschek realized the key was to have a very small diffusion region to permit a fast diffusion and thus a fast inflow. He set up a situation which includes a small Sweet–Parker diffusion layer at its centre from which four magnetoacoustic shocks extend from each corner out into a slightly curved external magnetic field that lies oppositely directed on either side of the diffusion layer (figure 3). He assumed the reconnection is both steady and incompressible and calculated its maximum reconnection rate to be

$$\max M_{\mathrm{Ae}} = \frac{\pi}{8 \log R_{\mathrm{me}}} \approx 0.1.$$

Such a rate of reconnection is fast enough to explain the energy release in solar flares and so is classed as fast reconnection.

(*d*) *After Petschek*

After Petschek announced his fast reconnection regime, the solar physics community sat back happily and thought that a key feature of solar flares had been explained. However, in 1986, disaster struck! Biskamp (1986) published a simulation of numerical reconnection in which he failed to find Petschek's fast reconnection



(*a*) slow compression ($b < 0$) (*b*) Petschek ($b = 0$)
(*c*) hybrid expansion ($0 < b < 2/\pi$) (*d*) hybrid expansion ($2/\pi < b < 1$)
(*e*) Sonnerup ($b = 1$) (*f*) flux pile-up ($b > 1$)

Figure 4. (*a*)–(*f*) Graphs showing examples from the almost-uniform family of reconnection regimes. Only the upper half of each magnetic configuration is shown.

mechanism and, therefore, claimed that Petschek reconnection did not exist. This shook the solar physics community and, indeed, for some the dust has never settled. Close inspection of Biskamp's assumptions, however, show that they are really quite different from Petschek's. In particular, the boundary conditions used by Biskamp are not the same as Petschek's and so, not surprisingly, when the equations are solved different solutions arise. This has been explained elegantly in a paper by Priest & Forbes (1986). They managed to generalize Petschek's theory and discovered a family of so-called almost-uniform reconnection regimes by including the effect of pressure and investigating different types of inflow. In figure 4, the upper halves of six different reconnection regimes are shown, each with a different inflow determined by the parameter b. The solid (horizontal) lines are magnetic field lines and the dashed (vertical) lines are streamlines. In figure 4*a*, $b < 0$ and the streamlines gently curve inwards towards the diffusion region, which is indicated by a small box at the bottom of each graph. In this case there is a compressional inflow. In figure 4*c*–*f*, $b > 0$ and the streamlines curve away from the diffusion region and so here there is expansion of the field. In figure 4*b*, $b = 0$ and Petschek reconnection occurs with neither expansion nor compression.

The rate of reconnection, M_{e}, for each of these six cases can be calculated for various different inflows, M_{i} (figure 5). Clearly, the rate of reconnection in Petschek's regime reaches a maximum as Petschek predicted. Similarly, the regimes with compressional inflow ($b < 0$) also have a maximum reconnection rate, although the maximum rates for these regimes are lower than the Petschek rate. However, regimes with $0 < b < 1$, called fast expansion regimes, can reach much faster rates than

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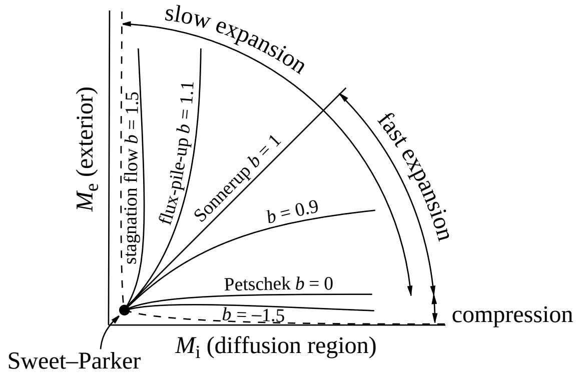

Figure 5. The dimensionless reconnection rate, M_e, versus the dimensionless inflow speed, M_i, for different reconnection regimes.

Petschek. When $b > 1$ the reconnection is said to be of slow expansion type and here the reconnection rate can be very fast; however, the reconnection process can be choked off for too great an inflow velocity.

Even though Biskamp's claim that Petschek reconnection does not exist cannot be proved from his simulations, his simulations brought to light several interesting phenomena. Reverse current spikes were found at the ends of the current sheet and strong jets occurred along the separatrices extending out from the current sheet. Both of these phenomena have been found in many other two-dimensional reconnection simulations and have even been reproduced in analytical models (Strachan & Priest 1993).

It has now been demonstrated numerically that Petschek's mechanism and the other fast reconnection regimes of Priest & Forbes (1986), Priest & Lee (1990) and Yan *et al*. (1992) do indeed exist when the resistivity is enhanced in the diffusion region. However, the focus has now largely turned towards investigating reconnection in three dimensions which, as we will see in the section below, is considerably more complex and interesting.

3. Three-dimensional reconnection

During the 1990s, attention increasingly focused on how reconnection occurs in three dimensions. This step is, of course, extremely important since space is three dimensional. In two dimensions, reconnection must take place at a null and can be defined as the transfer of flux across separatrices or as the process by which magnetic-field topologies are changed. These definitions are, however, not robust in three dimensions. For example, a two-dimensional null is equivalent to a neutral line in three dimensions, which is structurally unstable. This means that a new definition for reconnection in three dimensions has to be found. Various suggestions have been made by Schindler *et al*. (1988), Hesse & Schindler (1988), Priest & Démoulin (1995), Hornig & Schindler (1996) and Hornig & Rastätter (1998). What we do know, however, is that reconnection in three dimensions is far less constrained than in two dimensions and can occur at both three-dimensional nulls, as well as in regions where there are no nulls. What is now believed to be important for reconnection to occur is

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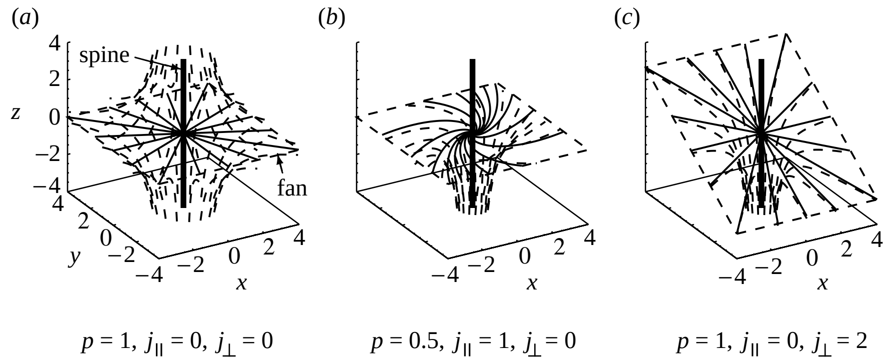


Figure 6. Linear three-dimensional nulls: (*a*) radial potential null. Non-potential nulls: (*b*) spiral null, with only current parallel to spine; (*c*) radial, with only current perpendicular to spine.

the presence of magnetic-field aligned electric fields along with some 'singular' field line together with an x-type topology in planes perpendicular to that field line. These electric fields can lead to diffusion of field lines and changes of magnetic connectivity (Priest & Forbes 1992; Hornig & Rastätter 1998).

In the following sections some of the theoretical mechanisms by which reconnection can occur in three dimensions are explained. First, however, the structure of three-dimensional null points is described.

(*a*) *Three-dimensional nulls*

In three dimensions reconnection can occur near nulls, but drawing direct analogies between two-dimensional reconnection at an X-point and three-dimensional reconnection near a null is not easy and probably not wise! The main reason is because a three-dimensional null is very different from a two-dimensional null (Cowley 1973; Fukao *et al*. 1975; Parnell *et al*. 1996).

In two dimensions, a null has just a pair of field lines, or strictly speaking four field lines, that go into the null and divide topologically distinct regions. Two are directed in and two are directed out from the null (figure 1).

In three dimensions, the equivalent features to the separatrices are the *fan* and *spine* of the null (Priest & Titov 1996). These features are sometimes called the Σ surface and γ line, respectively, though such names are not very intuitive (see, for example, Greene 1988; Lau & Finn 1990). The fan consists of an infinite number of field lines that enter (or leave) the null and make up a separatrix surface. The spine is made up of just two field lines that leave (or enter) the null. Furthermore, if all the field lines in the fan are directed away from the null, then the field lines in the spine are directed into it and the null is called a positive null. If the field lines in the fan are directed into the null, then in the spine they are directed away from the null and the null is said to be a negative null. Figure 6*a* shows a potential three-dimensional null with the spine shown in bold and the fan outlined by a dashed square. In all the figures the dashed field lines represent flux surfaces near the null. All potential nulls have their spine and fan perpendicular to one another.

In figure 6*b*, *c*, non-potential nulls are drawn. The first possesses just current parallel to the spine. Such a component of current does not move the spine and fan, but it does alter the structure of the field lines in the fan. For instance, in figure 6*b* the field lines in the fan are spiralled. The null depicted in figure 6*c* possesses just current perpendicular to the spine. Such a component of current causes the spine and fan to close up. Obviously, components of current both parallel and perpendicular to the spine cause both the fan to move and movement of field lines in the fan.

Now that we know something about the structure of three-dimensional nulls, we can start to consider how reconnection may occur in the locality of such nulls.

(*b*) *Reconnection near nulls*

To investigate how reconnection may occur near a single three-dimensional null, Priest & Titov (1996) took a kinematic approach and considered the global behaviour of the velocity, $\boldsymbol{v}$, and magnetic field, $\boldsymbol{B}$, in the vicinity of the null. Assuming that the system evolves in a steady and ideal manner through a series of equilibria with $\boldsymbol{j} \times \boldsymbol{B} = \boldsymbol{0}$, then Ohm's law and Faraday's law reduce to

$$\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \boldsymbol{0}, \tag{3.1}$$

$$\nabla \times \boldsymbol{E} = \boldsymbol{0}. \tag{3.2}$$

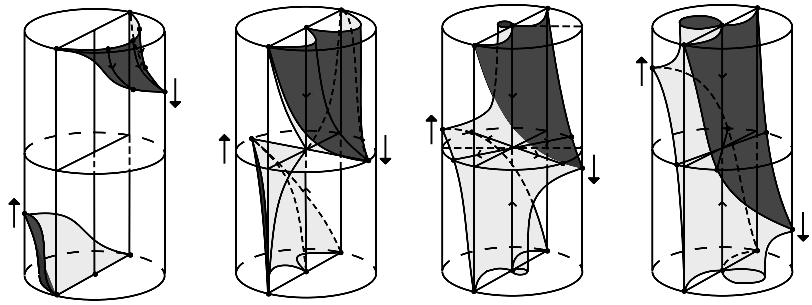
The magnetic field is prescribed in the entire region and the boundary conditions of a particular velocity flow are given. Ohm's law and Faraday's law are then solved for $\boldsymbol{v}$; in particular, singularities are sought in the flow. Singularities in the flow are important as they indicate that the assumption of ideal evolution of the field cannot be true in the location of the singularity and thus non-ideal effects, such as current enhancements, must be important, leading to reconnection at the singularities.

(i) *Spine reconnection*

Priest & Titov (1996) considered two examples. In the first, the magnetic field is taken to be that of a radial null such that $\boldsymbol{B} = (R, 0, -2z)$ in cylindrical polars. They then imposed a flow along the curved surface of a cylinder of radius $R = 1$. The flow was such that the footpoints of a flux surface cutting the cylinder were driven downwards from the $z = 1$-plane to the $z = -1$-plane across the fan of the null for $0 < \theta \leqslant \pi$ and upwards from the $z = -1$-plane to the $z = 1$-plane, again across the fan, for $\pi < \theta \leqslant 2\pi$. Figure 7 shows the evolution of two flux surfaces under such conditions. As the footpoints of the field lines in the flux surfaces are swept down (up) the curved surface of the cylinder, the other ends of the field lines that cut the $z = 1$ ($z = -1$) plane are swept in towards the spine. At the spine the velocity becomes singular, implying that non-ideal effects become important and that reconnection can occur. The reconnection allows the flux surface to bubble through to the other half of the cylinder. Such a process has been named *spine reconnection*.

(ii) *Fan reconnection*

In the second example the magnetic field is once again taken to be that of a radial null. However, this time the flow imposed is along the surfaces $z = 1$ and $z = -1$ at either end of the cylinder. Such a flow drives the footpoints of field lines, lying in flux surfaces, across the spine. The flux surfaces themselves drop down towards the fan plane and swing across like curtains (figure 8). A singularity in



(*a*) (*b*) (*c*) (*d*)

Figure 7. Sketch of spine reconnection. The spine runs down the centre of the cylinder and the fan lies in the horizontal plane that bisects the cylinder.

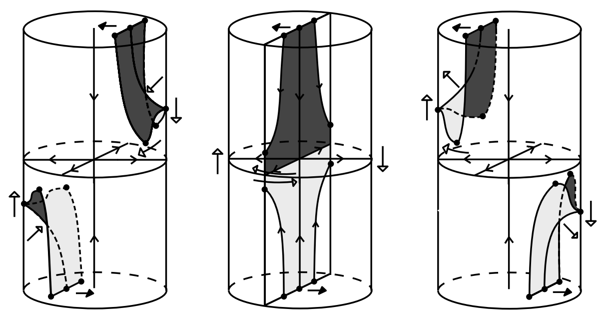


Figure 8. Sketch of fan reconnection. The spine runs down the centre of the cylinder and the fan lies in the horizontal plane that bisects the cylinder.

the velocity is produced in the fan plane and, as before, this produces non-ideal effects that allow reconnection to occur. This type of reconnection is known as *fan reconnection*.

(iii) *Reconnection at multiple nulls*

The two types of reconnection described above occur in isolated single nulls. However, evidence from magnetic-field configurations with many different fragments suggest that nulls, in general, occur in pairs—a negative null paired with a positive null. It is, therefore, likely that reconnection will actually occur simultaneously at both of these nulls (Lau & Finn 1990; Galsgaard *et al*. 1997). The above examples, however, do illustrate the importance of the spine and fan as sites of reconnection. Indeed, it is found that when reconnection occurs at a double null these special field lines once again play an important role. The only major change that occurs for reconnection at a double null is that the reconnection occurs preferentially along the special field line that lies in the fan or spine of each null and connects the two nulls. Such a line is called a *separator*.

The work on reconnection at three-dimensional nulls is still in its early years and there is a long way to go before we properly understand everything about it. Interestingly though, many people, in particular those working on numerical simulations of magnetic field behaviour, are beginning to suggest that reconnection in the absence

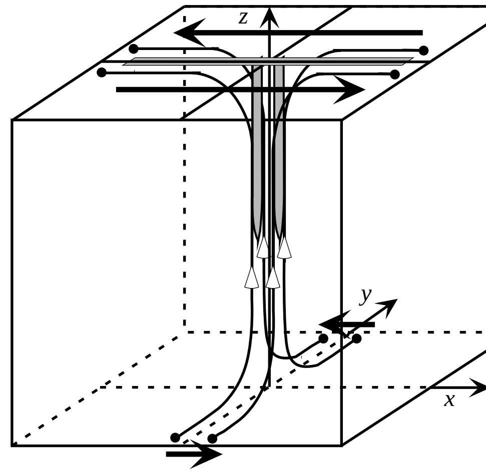


Figure 9. Sketch of reconnection at a quasi-separatrix layer.

of nulls may be just as important as reconnection at nulls, since strong field-aligned electric fields can be produced without them. In the next section, we show such an example.

(*c*) *Reconnection in the absence of nulls*

After careful study of many flare sites on the Sun and numerous reconstructions of the magnetic-field topology at such sites, Démoulin and co-workers came to the conclusion that there were very few nulls above the surface of the Sun. They, therefore, thought it was unlikely that null-point reconnection was the answer to solar flares and so set out to find a new mechanism. The resulting work by Priest & Démoulin (1995) suggests a possible mechanism for reconnection at regions called *quasi-separatrix layers*. They use a similar kinematic approach to that used by Priest & Titov (1996).

The magnetic field imposed by Priest & Démoulin (1995) is of the form

$$\boldsymbol{B} = (x, y, -l), \tag{3.3}$$

where $l \ll 1$. Clearly, such a field is both three dimensional and non-zero everywhere. A typical field line in this field has the form

$$(x, y, z) = (x_0 \mathrm{e}^{z/l}, y_0 \mathrm{e}^{-z/l}, z).$$

So, if the footpoint of a field line in the $z = 0$-plane is $(x_0, y_0, 0)$, its endpoint in the $z = 1$-plane would be $(x_0 \mathrm{e}^{1/l}, y_0 \mathrm{e}^{-1/l}, 1)$. If the footpoint of this field line moved from

$$(x_0, y_0, 0) \to (-x_0, y_0, 0),$$

then its endpoint would move from

$$(x_0 \mathrm{e}^{1/l}, y_0 \mathrm{e}^{-1/l}, 1) \to (-x_0 \mathrm{e}^{1/l}, y_0 \mathrm{e}^{-1/l}, 1).$$

The footpoint of the field line merely moves a distance $2x_0$, but the endpoint moves a distance $2x_0 \mathrm{e}^{1/l} \gg 2x_0$ since $l \ll 1$. This implies that the field line may move at

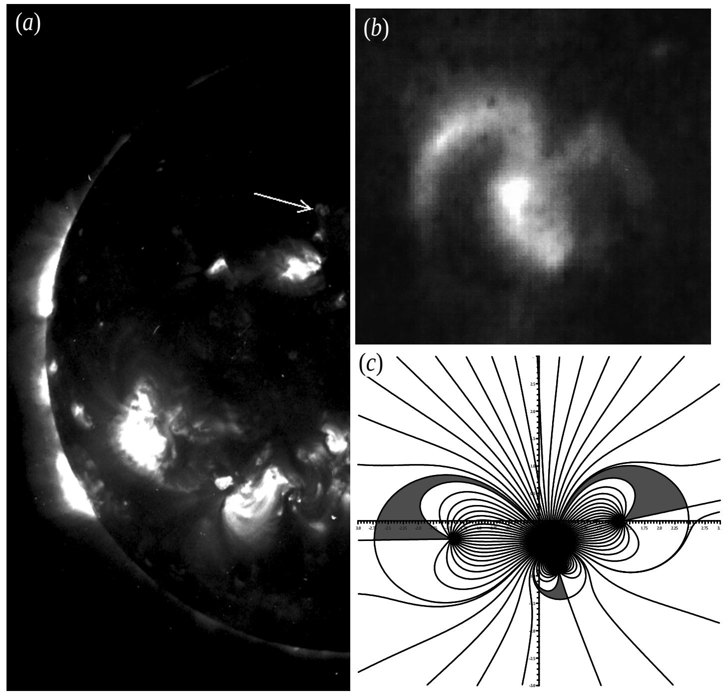


Figure 10. (*a*) Half of a full-disc soft X-ray image of the corona (NIXT, Golub) with arrow pointing at bright point. (*b*) Enlargement of bright point. (*c*) Sketch of field lines in bright point—area of newly reconnected field lines shaded.

a speed faster than the Alfvén speed and hence the field lines may slip through the plasma. The regions where the field line speed becomes super-Alfvén are known as *quasi-separatrix layers* and they are the regions where reconnection takes place. In figure 9, the two solid lines represent the two positions of the field line discussed above and the quasi-separatrix layers are indicated by shaded regions.

Further theoretical studies have been carried out on quasi-separatrix layers by Inverarity & Titov (1997) and Galsgaard (2000). Also, Mandrini *et al.* (1996) have used the theory of quasi-separatrices to explain the heating in an observed coronal bright point in the solar corona.

4. Applications of reconnection to coronal heating

As already discussed, magnetic reconnection can occur in many astrophysical phenomena; however, in this section, I will just consider the application of the above reconnection theories to solar phenomena. The solar corona plays host to many small dynamic events, such as coronal bright points and microflares, as well as containing many larger phenomena such as active regions and solar flares. Reconnection almost certainly plays a major role in creating and maintaining all of these features. One particular phenomenon, coronal bright points, provides some excellent examples of events that have been best explained by reconnection.

(*a*) *Coronal bright points*

Coronal bright points are small-scale bright events that can be observed in both X-ray and UV lines (figure 10*a*). They have diameters of up to 3×10^7 m and lifetimes from 0.1 h up to 48 h. Typically, they release between 10^{18} and 10^{21} J of energy over their complete lifetime. They are found distributed throughout the quiet corona and coronal holes and are observed to appear above pairs of opposite-polarity magnetic fragments. Originally, it was thought that coronal bright points were a result of emergence of new flux (Golub *et al*. 1974); however, it has now been appreciated that many (greater than 70%) of these opposite-polarity pairs are actually cancelling magnetic features (Harvey 1985). That is to say, bright points are usually associated with magnetic fragments that are converging and mutually reducing in flux (Martin 1984). A model, called the converging flux model, for coronal bright points and associated cancelling magnetic features by Priest *et al*. (1994) is described below.

Initially, in the photosphere two opposite-polarity magnetic fragments are situated in an overlying field such that they are magnetically unconnected. This is in line with evidence from observations and is called the *preinteraction phase* in the model (figure 11*a*, *b*). The two fragments are slowly driven towards one another by the convection motions in supergranule cells. The driving is sufficiently slow that the field evolves through a series of quasi-static equilibria. Eventually, the two fragments will come close enough to start interacting and a null will form in the photosphere. As the fragments continue to be driven together reconnection occurs at the null as it rises into the corona (*interaction phase*) (figure 11*c*, *d*). The release of energy at the null gives rise to the bright point with hot dense plasma being ejected along the newly reconnected field lines. This stage draws to a close as the null drops back down to the photosphere signalling the start of the *cancellation phase* where reconnection occurs in the photosphere as the two fragments finally cancel with one another (figure 11*e*, *f*).

Clearly, such a mechanism can occur on many scales from very small fragments up to large fragments. Thus, it is highly probably that this mechanism produces events of all sizes from *ca.* 10^{16} J or smaller up to 10^{21} J or more. The amount of energy released and the way in which energy is released depends not only on the size of the fragments, but on the strength of the overlying field and the rate at which the fragments are being driven together. For instance, if the driving were not at all steady and the reconnection were of the flux-pile-up type then the event may appear to blink on and off; or with a different driving and more steady reconnection the event may last for several hours.

The principles of this model have been extended into three dimensions to model two observed bright points (Parnell *et al*. 1994). One of the observed bright points is shown in figure 10*b* with the corresponding observed magnetic field in the photosphere drawn in figure 10*c*. This bird-shaped bright point was associated with one positive and three negative magnetic fragments. Observed motions of these fragments indicate that, to form the three observed bright loops of the bright point, spine reconnection must have occurred at the three neutral points in the magnetic field. Other observed bright points have also been explained by reconnection; for instance, van Driel-Gesztelyi *et al*. (1996) describe the heating of a bright point by reconnection at a double null.

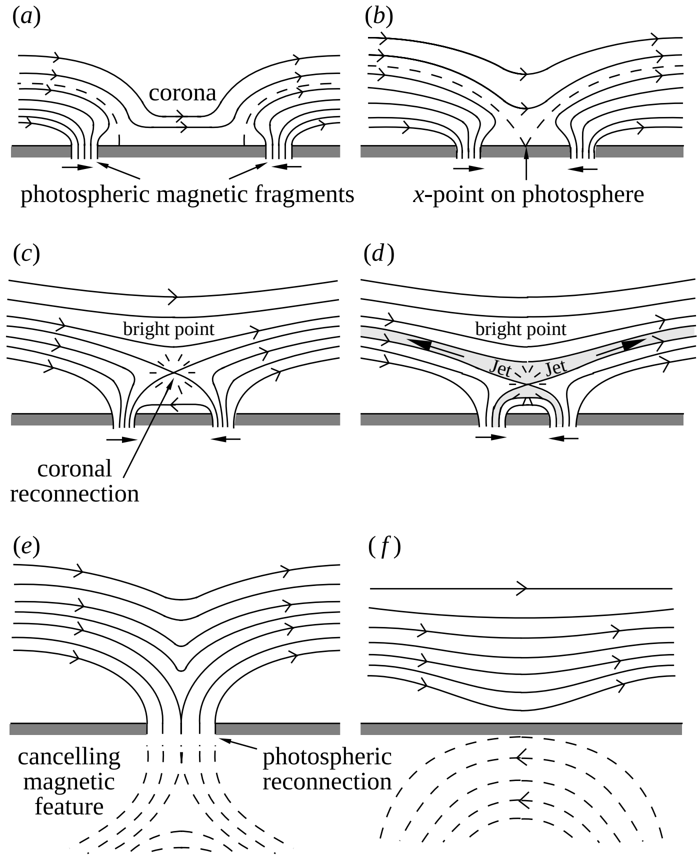


Figure 11. Converging flux model (qualitative): (*a*) and (*b*) preinteraction phase, where fragments are unconnected; (*c*) and (*d*) interaction phase, in which reconnection creates a bright point; (*e*) and (*f*) cancellation phase, where photospheric reconnection forms cancelling magnetic features.

(*b*) *Nanoflare and microflare heating of the corona*

Detailed studies of the complex magnetic fields in the solar photosphere reveal that there is a continuous random pattern of positive and negative magnetic fragments covering the entire solar photosphere. This magnetic coverage of the entire surface of the Sun has been called the *magnetic carpet* (Schrijver *et al*. 1998). The magnetic fragments of the carpet occur on large scales, such as sunspots, as well as very small scales equivalent to instrument resolution and probably smaller. The carpet pattern is not stationary, but is continuously changing through emergence of new flux, cancellation of existing flux, fragmentation of large flux regions and merging of smaller regions. Indeed, in the quiet Sun, the total amount of flux that emerges in a 40 h period is equivalent to the entire flux in the quiet Sun. Since the total flux remains at a constant level of *ca.* 3×10^{15} Wb, this means that the rate of cancellation must be equivalent to the emergence rate (Schrijver *et al*. 1998). Clearly, with all this mixing, emerging and cancelling, the links between magnetic fragments must be continuously changing. This implies that there must be reconnection occurring at all times above the quiet magnetic carpet. Although these multiple reconnections are

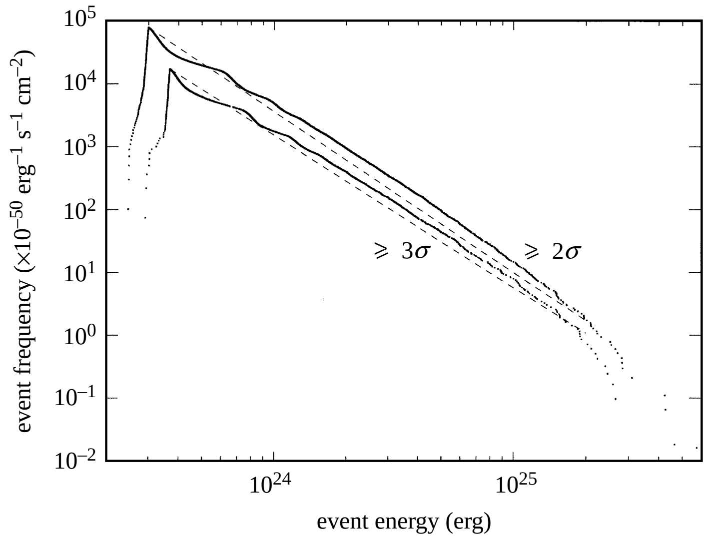


Figure 12. Frequency of event versus event energy in the quiet Sun.

very small, one would expect them to release some energy. The key question is, do they release enough energy to explain the observed losses from the quiet solar corona?

April 1998 saw the launch of TRACE, the Transition Region And Coronal Explorer, a new Earth-orbiting solar telescope (Schrijver *et al.* 1999). It has an angular resolution of 1 arcsec (726 km) and a cadence of *ca.* 40 s for the quiet coronal lines. Using TRACE data of a quiet region of the corona, Parnell & Jupp (2000) studied events with energies between 4×10^{16} and 10^{18} J (4×10^{23} and 10^{25} erg), i.e. events with nanoflare energies. They find that the frequency of events follows that of a power law with index between -2.4 and -2.6, if a constant line-of-sight depth is assumed (figure 12), and between -2.1 and -2.0, if a line-of-sight depth proportional to the square root of the area is assumed. Thus, since the index of the power law is less than -2, the total power from these small events dominates that from large events. However, extrapolating the power law beyond 10^{16} J (10^{23} erg) implies that events at least as small as 10^{13} J (10^{20} erg) are required to heat the quiet solar corona.

Parnell & Jupp (2000) also investigated the spatial distribution of these events and found that they occurred, in general, at the edges of supergranule cells, the most likely location of magnetic cancellation in the photosphere. This would suggest that the energy release site is near the footpoints of loops. However, they also discovered evidence for energy releases along loops and, therefore, directly in the high corona. In the 24 min period of their observations they found that just 16% of the quiet corona had at least one event in the observing period. Clearly, therefore, the energy distribution in the quiet corona is in no way uniform, but is localized in millions of events of all different shapes and sizes.

5. Conclusions

Over the last 50 years the important fundamental plasma-physics process called magnetic reconnection has been discovered and investigated at great length. We now understand two-dimensional reconnection fairly well on a global scale, but there is

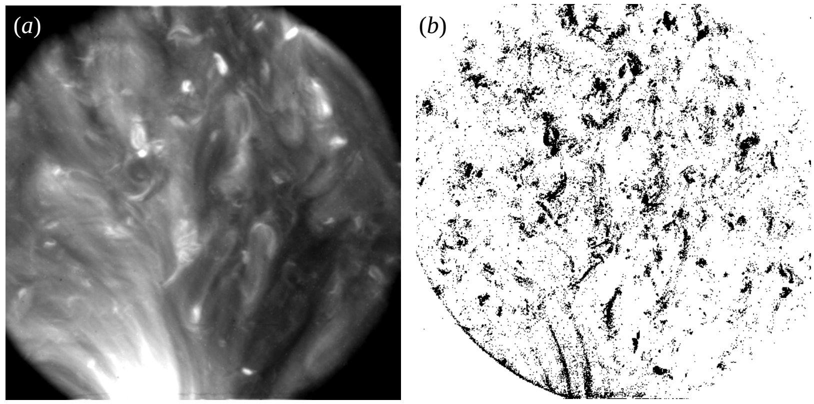


Figure 13. (*a*) An Fe XII image of a quiet-sun region taken by TRACE at 20:40 h on 16 June 1998. (*b*) The spatial distribution of events with enhancements at least 2σ in size and with one or more pixel.

still much to learn about the microscopic plasma physics involved in this process. Progress has also been made in understanding the complexities of reconnection in three dimensions. There have been several mechanisms proposed to date for three-dimensional reconnection both at nulls and in the absence of nulls. For instance, at a single null, reconnection can occur along the spine of the null or across the fan surface. In situations where there are multiple nulls, separator reconnection is more likely to occur involving simultaneous reconnection at two nulls. Reconnection can also occur in the absence of nulls in three dimensions, for instance, at quasi-separatrix layers. The important thing we have learnt about all these types of reconnection is that none of them are exactly analogous to two-dimensional X-type reconnection. This means that, although all our studies of reconnection in two dimensions have been invaluable to our understanding of reconnection, it is important that we now move on and consider in much greater detail the fundamentals of reconnection in three dimensions.

Several models of observed phenomena have been explained by three-dimensional reconnection. For example, different observed coronal bright points have been modelled by both reconnection at nulls and in the absence of nulls. Reconnection at the Earth's dayside magnetopause has been successfully modelled in three dimensions, as have substorms at the magnetotail.

Magnetic reconnection is, clearly, a very important plasma process. It occurs in both large-scale phenomena, like solar flares and coronal mass ejections and also on small scales. Indeed it is possible that the solar corona is powered by a multitude of small-scale events with energies as small as 10^{14} J which are most likely to be powered by magnetic reconnection.

With only half a century's work so far on reconnection we have made great strides in understanding this process, but at least one, if not more, half century of focused work is likely to be needed before all the key secrets about magnetic reconnection are known.

I thank the organizing committee of The Royal Society discussion meeting for inviting me to talk. Also, I thank all those that questioned me afterwards. Their inquiries have both consolidated my thoughts and stimulated new ideas. Finally, I thank the Royal Astronomical Society for their continued support of my fellowship.

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Discussion

D. Gough (*University of Cambridge, UK*). How does that mass flux compare with the mass flux in the solar wind? Does most of the mass going up come down again or is a good fraction of it blown away?

C. E. Parnell. Gravity helps the plasma to drain down a magnetic loop that is rising. Certainly, in emerging-flux regions you always see downflows along the legs. My impression is that there is substantial upflow and downflow between the photosphere and corona and that the majority of the plasma in a photospheric flux tube must drain down before a flux tube escapes into the solar wind.

L. Golub (*Smithsonian Astrophysical Observatory, USA*). On TRACE movies you can see upflows almost everywhere in the corona and certainly there are places where they are very obvious and very clear—material flowing upwards at about what looks like the local sound-speed. There are lumps in the flow and we're trying to find out now if they are associated with reconnection events down in the boundaries between the network elements. There's a good chance that they are.

R. Hide (*Oxford, UK*). I wanted to ask about your models of reconnection. To what extent do the Hall currents have any significance on any of the scales or phenomena you've talked about?

C. E. Parnell. Hall currents possibly start to become important when the density becomes very low in the high corona. In the low corona they're not important. They've certainly not been considered in any great detail in coronal studies so far. The other part of the Sun where they're important is in the temperature-minimum region. Ambipolar diffusion may allow the transport of mass across field lines in the photosphere, in sunspots and intense flux tubes.

R. Von Fay Siebenburgen (*University of Sheffield, UK*). You have mentioned nanoflares and microflares, but you did not mention explosive events?

C. E. Parnell. Basically, I have not distinguished between nanoflares, microflares and explosive events. The time cadence that I have here is just less than 2 min. I categorized an event as an enhancement in emission measure, which can occur either through evaporation of new plasma entering in to a pixel or through the heating of *in situ* plasma.

E. R. Priest (*University of St Andrews, UK*). Can I just say for the non-solar people that so-called 'explosive events' are observed in one of the instruments on SOHO called SUMER. They show up as little jets of plasma going out in two directions up to 50–100 km s^{-1} at high spatial resolution (1 arcsec). It is thought that these little explosive events are the jets coming from reconnection sources between them. They tend to be located above the network, the boundaries of the large convection cells (supergranules), at locations where magnetic fields are interacting with one another. I would expect to find many of your TRACE brightenings to be associated with explosive events if we looked at them with SUMER at the same time.

Y. Uchida (*University of Tokyo, Japan*). We are also trying to do three-dimensional simulations back in Tokyo and I'm just wondering how you incorporate the three-dimensional characteristics into the theory?

C. E. Parnell. Certainly, for the bright points that I've looked at we have developed a three-dimensional model as well as the two-dimensional one I described. In place of the separatrix curves in two dimensions, in three dimensions you find domes—separatrix surfaces—that intersect in special field lines called 'separators'. These in turn end in three-dimensional null points. The bright points I have investigated appear to transfer flux across a fan surface by spine reconnection.

In future I hope to study nanoflare events from TRACE in more detail to try and determine more about their relation to the magnetic field and their cause.

E. R. Priest. It would be interesting if Alan Title were able to let us look at events at a factor of 10 smaller in energy, as I'm sure he will in the future. Would those extremely tiny events that are actually heating the corona also be distributed around supergranules or would they be located along large-scale loops? Would the energy release be low-down in the network or would it be high up in the corona?

R. Kerr (*NCAR, Boulder, USA*). From our recent numerical experiments on singularities in ideal three-dimensional MHD we see configurations, similar to your quasi-separatrix layers (QSLs). Maybe there is a large zoo, or maybe another class of reconnections besides what you were presenting?

C. E. Parnell. Yes, it is very likely that there are more classes of three-dimensional reconnection that I have not mentioned. Klaus Galsgaard from St Andrews has been investigating QSLs.

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K. Galsgaard (*University of St Andrews, UK*). The important feature that I have found with three-dimensional reconnection is that you have to have a stagnation flow and it has to be imposed by the magnetic field itself to generate a current sheet.

E. R. Priest. It's quite clear that we've only just started a huge voyage of discovery on studying three-dimensional reconnection; there are a lot of aspects that we've only just started to investigate. So far, certainly the implications are that you need small-scale local stagnation-point flows near the reconnection site. It's a huge topic. Terry Forbes and I have just sent a book off to Cambridge University Press on magnetic reconnection; there are 580 pages in it and it's being edited at the moment. I'm glad I'm not a copy-editor but there are 60 pages in there on three-dimensional reconnection, so if you are rich enough to buy it, it's coming out later on this year.

I suspect we're going to find lots of different types of reconnection regime. Gunnar Hornig is doing some fantastic work on a wide variety of three-dimensional reconnection regimes. There is an anti-reconnection theorem, which implies that the simplest approaches just won't work, and it means that the whole analysis technically is very complex.

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